CHAPTER 3

Block Diagrams & Signal flow graphs

By

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Signal flow Graphs

Signal-Flow Graphs

What is Signal Flow Graph (SFG)?

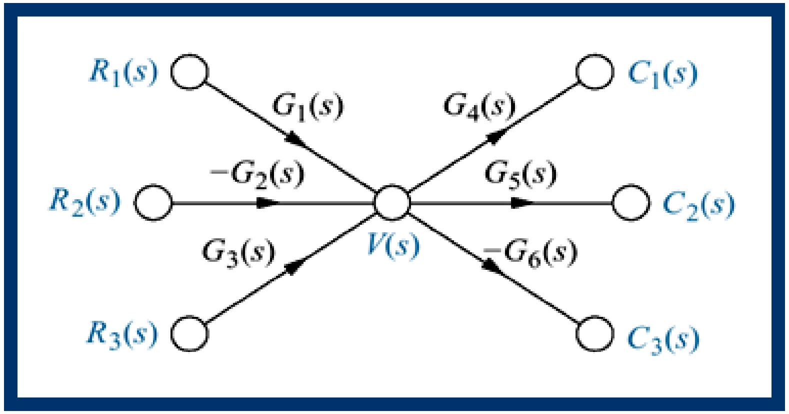
- SFG is a diagram which represents a set of simultaneous equations.
- This method was developed by Samuel Jefferson Mason. this method does not require any reduction technique.
- It consists of **nodes** and these nodes are connected by a directed lines called **branches**.
- Every branch has an arrow which represents the flow of signal.
- For complicated systems, when Block Diagram (BD) reduction method becomes tedious and time consuming then SFG is a good choice.

Signal-Flow Graphs

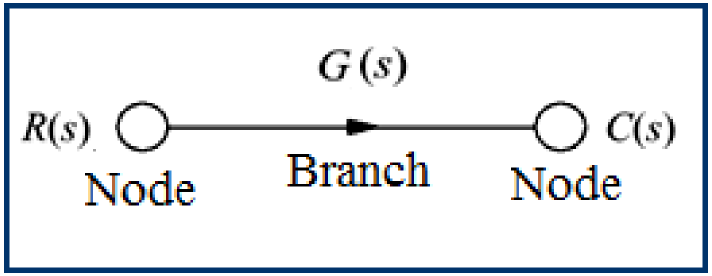
- Signal-flow graphs are an alternative to block diagrams, it consists only of branches and nodes.

Branches, which represent system variables

Nodes, which represent signals.



Signal-Flow Graphs



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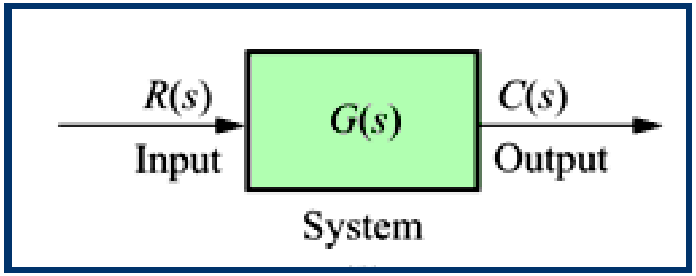
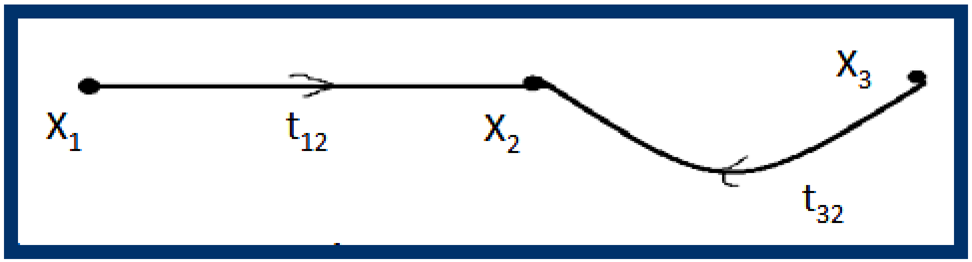


Signal-Flow Graph **Block Diagram**

Signal-Flow Graphs

Definition of terms required in SFG

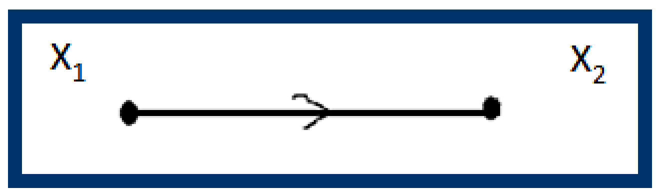
Node: It is a point representing a signal.



In this SFG there are 3 nodes.
i.e., 3 signals: X_1, X_2, X_3

In this SFG there are 2 branches.
i.e., 2 variable: t_{12}, t_{32}

Branch: A line joining two nodes.



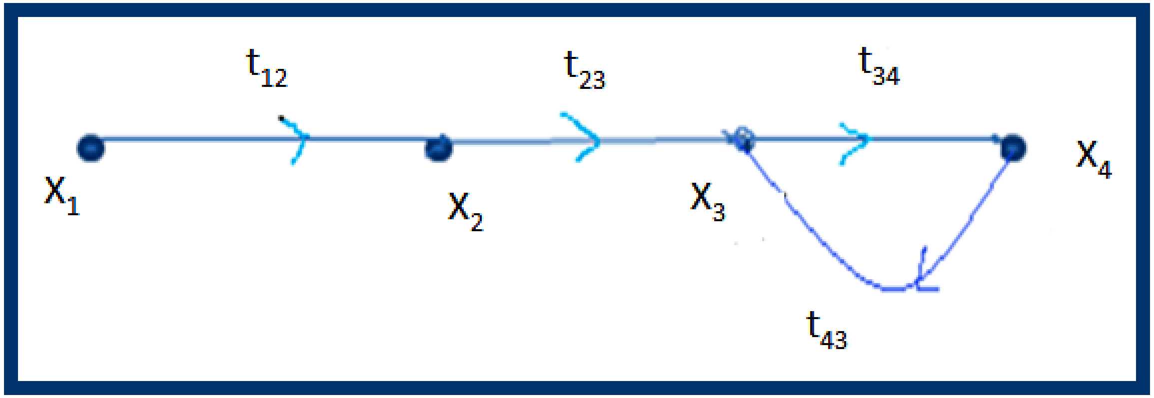
Input Node (Source): is a node that has only outgoing branches.
X_1 is input node.

Signal-Flow Graphs

Output node (sink): is a node that has only incoming branches.

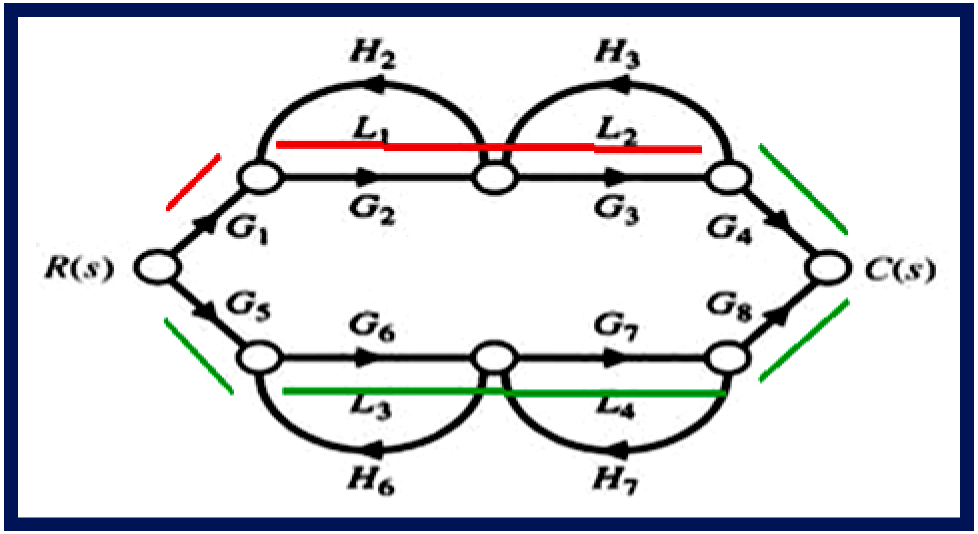
Mixed nodes: Has both incoming and outgoing branches.

Transmittance (Gain): It is the gain between two nodes. It is generally written on the branch near the arrow.



Signal-Flow Graphs

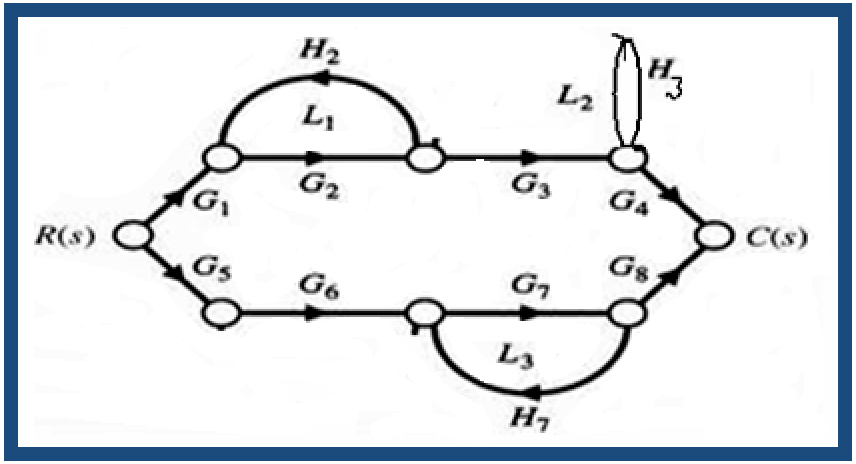
- **Path**: is a continuous connection of branches from one node to another with arrowhead in the same direction.
- **Forward path**: is a path connects (input) source node to (output) a sink node.
- **Forward Path gain**: It is the product of branch transmittances of a forward path.



$P_1 = G_1 G_2 G_3 G_4$, $P_2 = G_5 G_6 G_7 G_8$

Signal-Flow Graphs

- **Loop** : Path that starts and finished at the same node
- **Loop gain**: it is the product of T.F of all branches that form loop.
- **Non-touching loops**: Loops that don't have any common node or branch.



$L_1 = G_2 H_2$ $L_2 = H_3$

$L_3 = G_7 H_7$

Non-touching loops are:

L_1 & L2 , L_1 & L_3 , L_2 & L_3

Signal-Flow Graphs

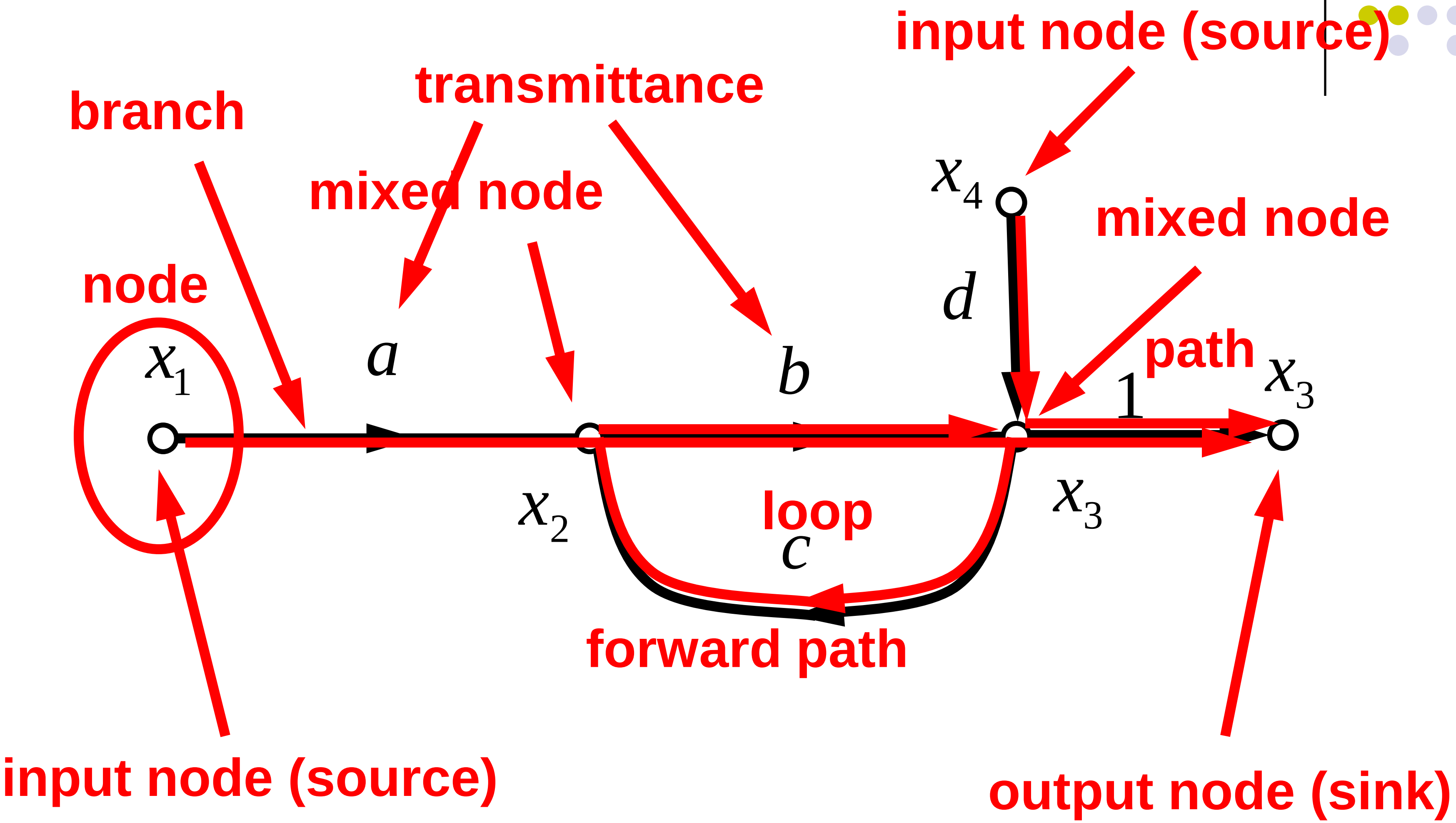
Rules for drawing of SFG from Block diagram

- All **signals**, **summing points** and **take off points** are represented by nodes.

- If a **summing point** is placed before a **take off point** in the direction of signal flow, in such a case the summing point and take off point shall be represented by a single node.

- If a **summing point** is placed after a **take off point** in the direction of signal flow, in such a case the summing point and take off point shall be represented by separate nodes connected by a branch having transmittance unity.

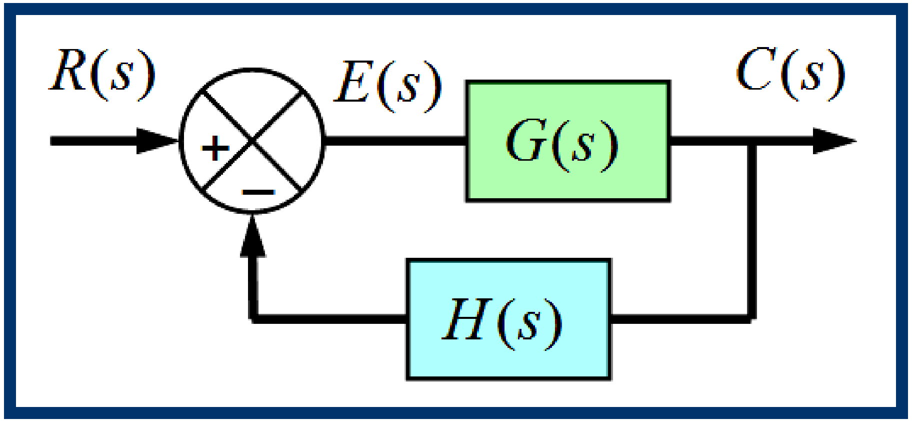
signal flow graph



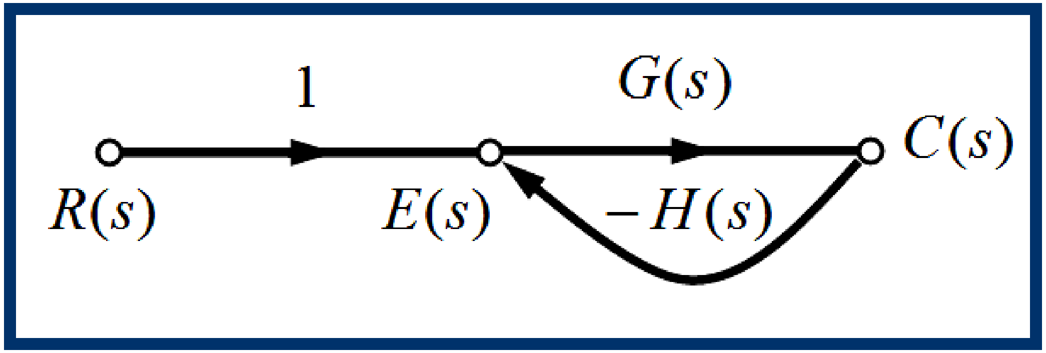
Comparison between Block Diagram and SFG

Signal-Flow Graphs

Block diagram

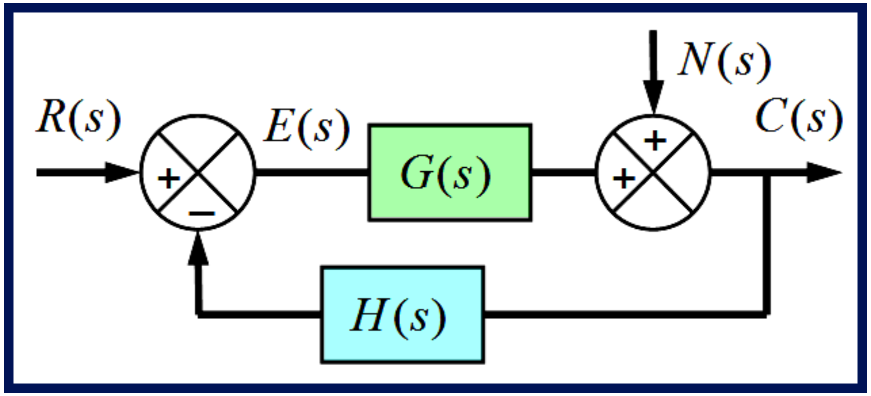


Signal flow graph

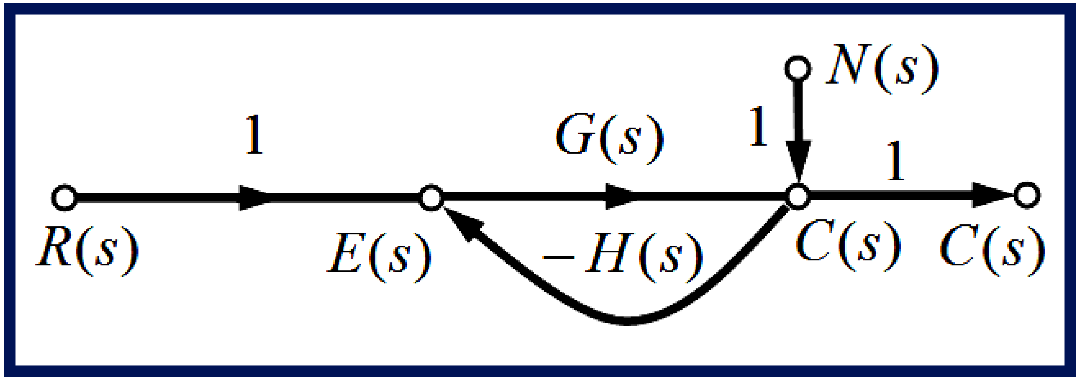


Signal-Flow Graphs

Block diagram

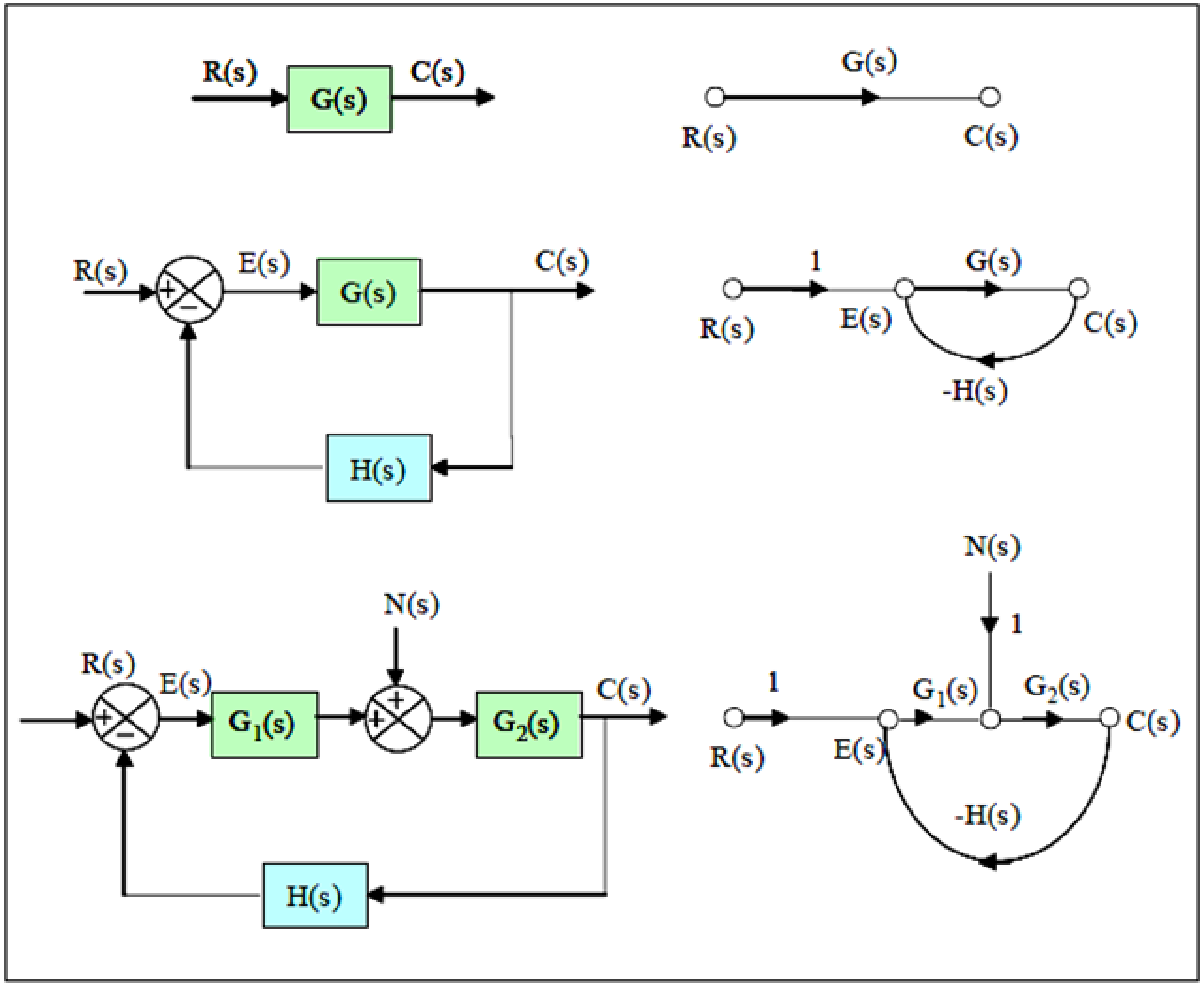


Signal flow graph



Signal-Flow Graphs

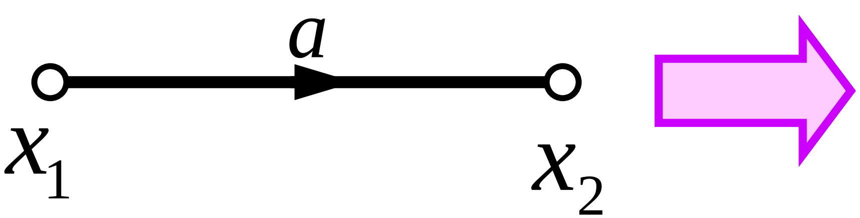
- **Examples of BD to SFG transformation**



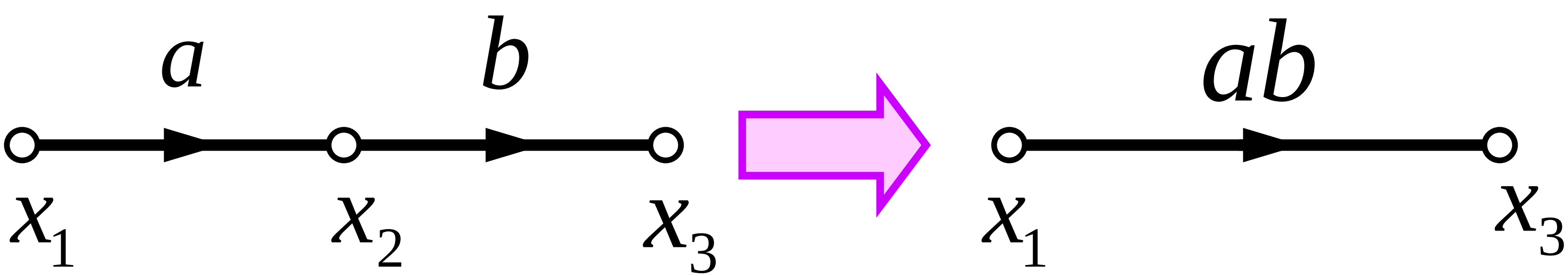
Signal flow Graph Algebra

Signal-Flow Graphs

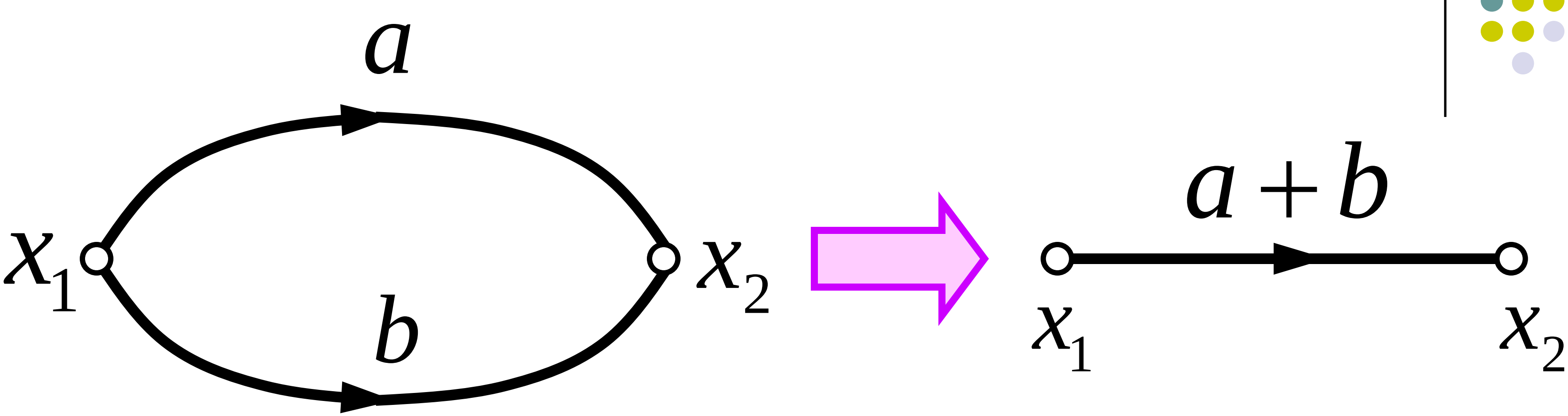
Signal Flow Graph algebra

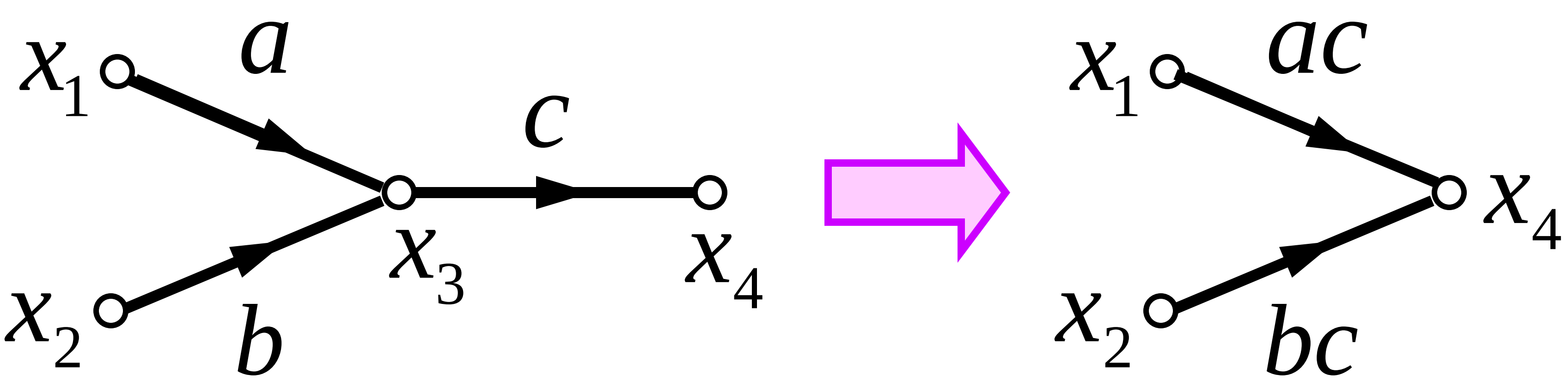


$$x_2 = a x_1$$

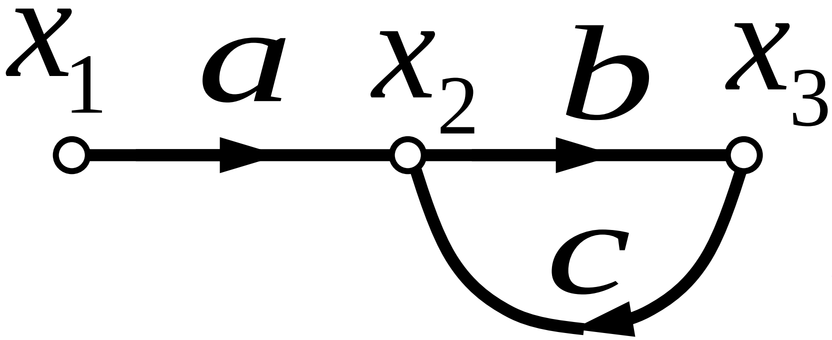


Signal-Flow Graphs



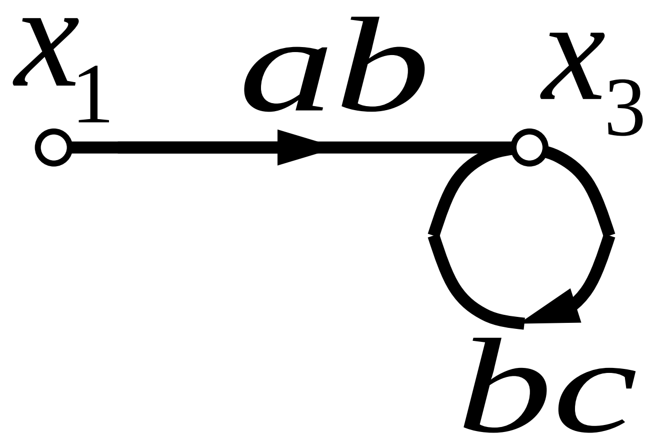


Signal-Flow Graphs

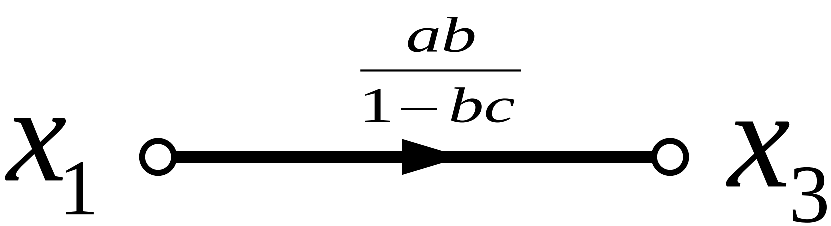


$$x_3 = bx_2$$

$$x_2 = ax_1 + cx_3$$



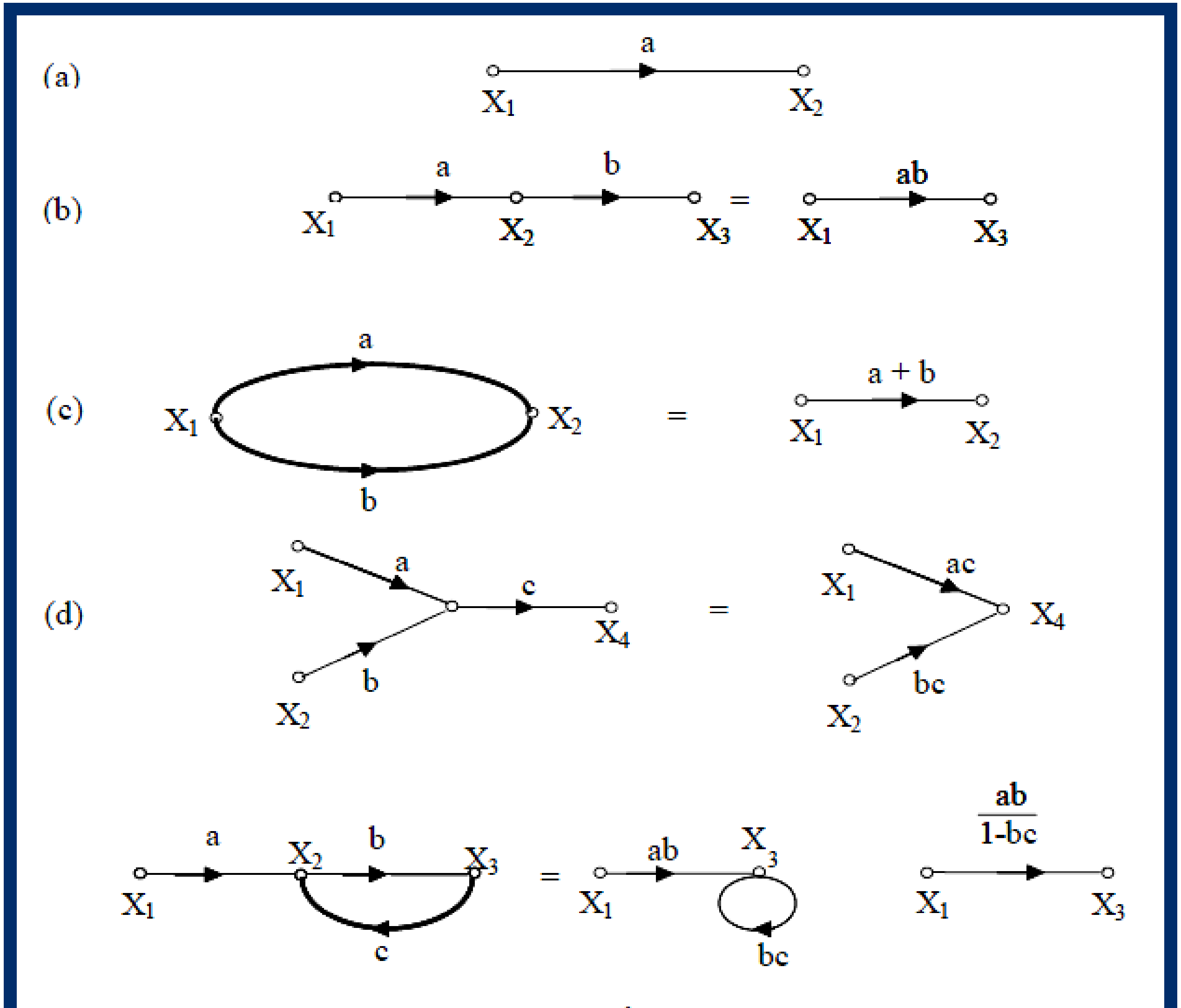
$$x_3 = abx_1 + bcx_3$$



$$\frac{x_3}{x_1} = \frac{ab}{1-bc}$$

Signal-Flow Graphs

- Basic rules with SFG transformation



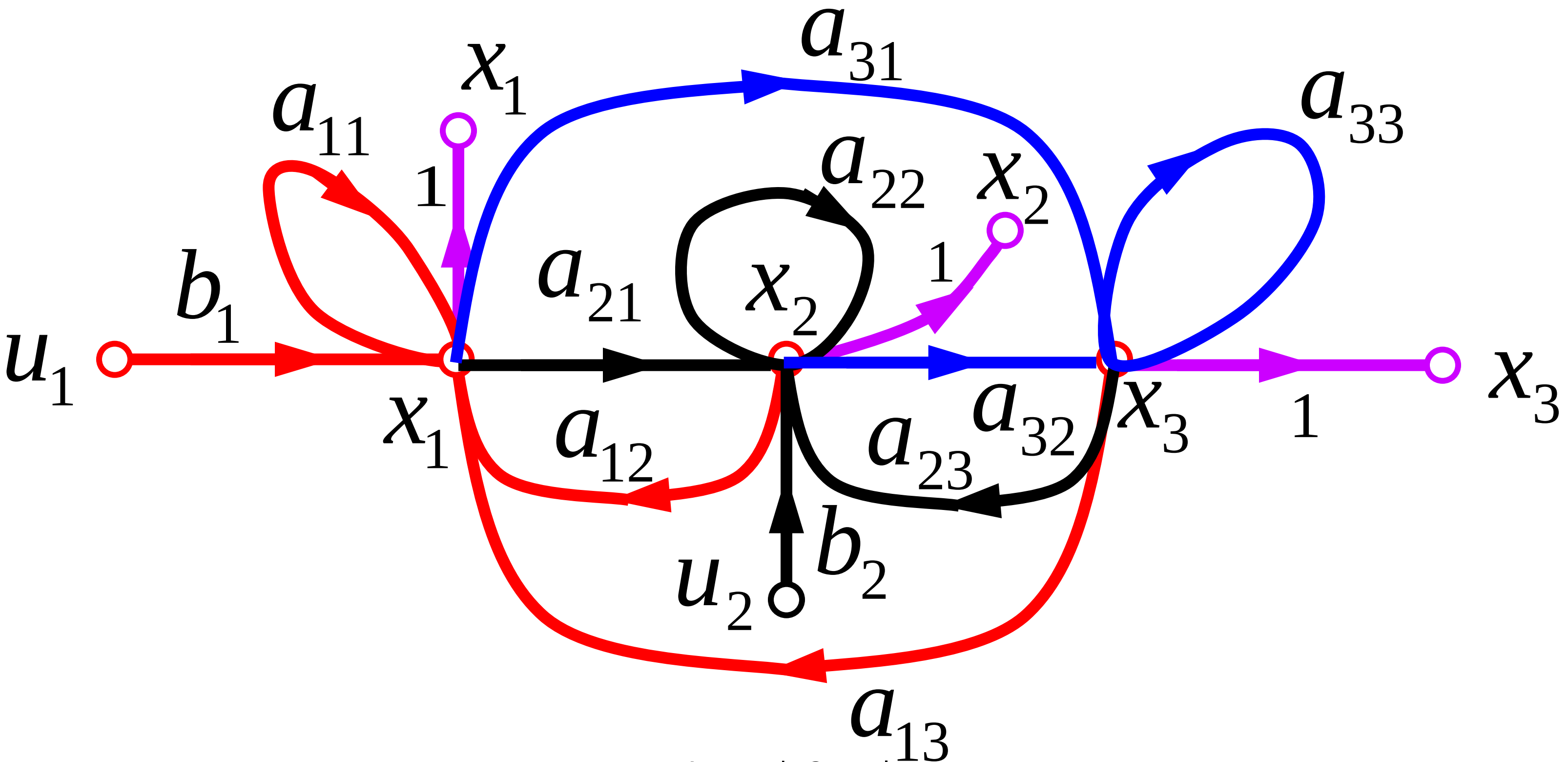
Signal-Flow Graphs

Signal flow graphs of linear systems

$$x_1 = a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + b_1u_1$$

$$x_2 = a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + b_2u_2$$

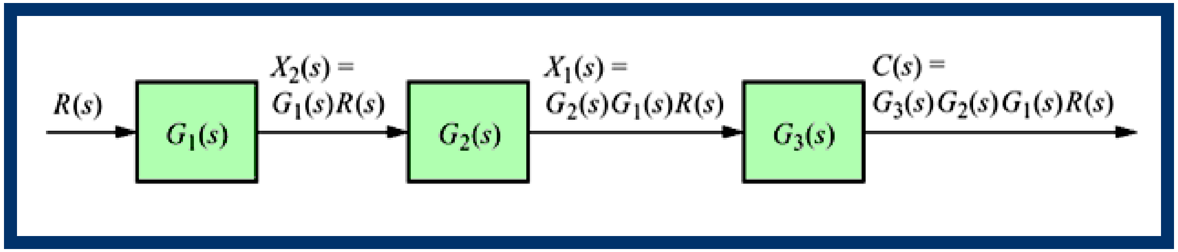
$$x_3 = a_{31}x_1 + a_{32}x_2 + a_{33}x_3$$



Example 1

Convert the cascaded, parallel, and feedback forms of the block diagrams shown in Figures, respectively, into signal-flow graphs.

1



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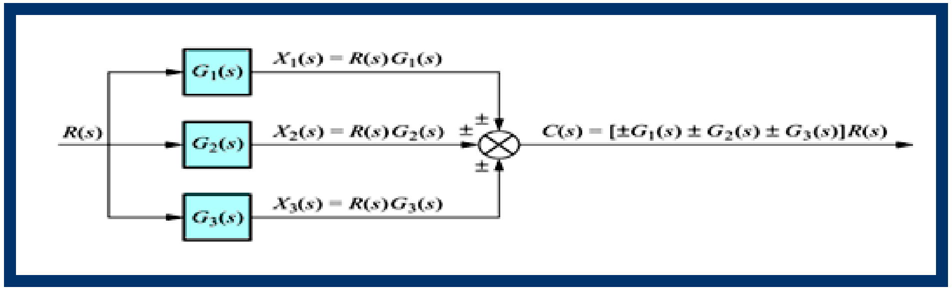

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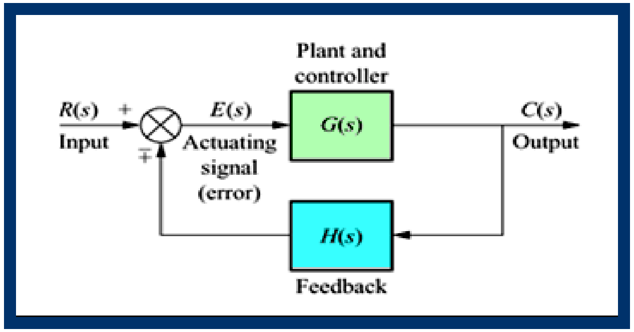
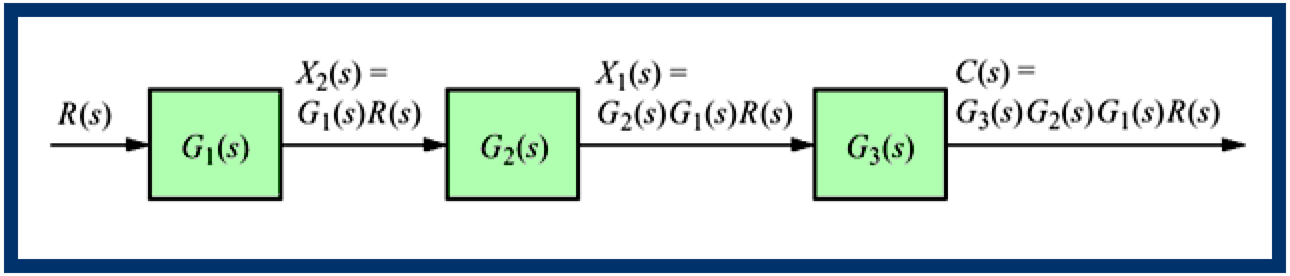


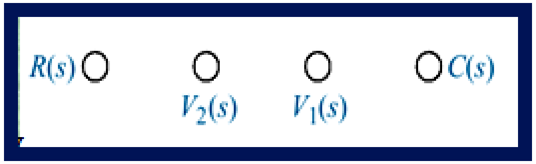
Example 1 (cont'd)

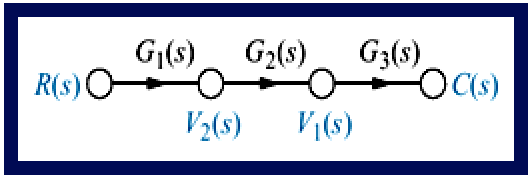


Solution

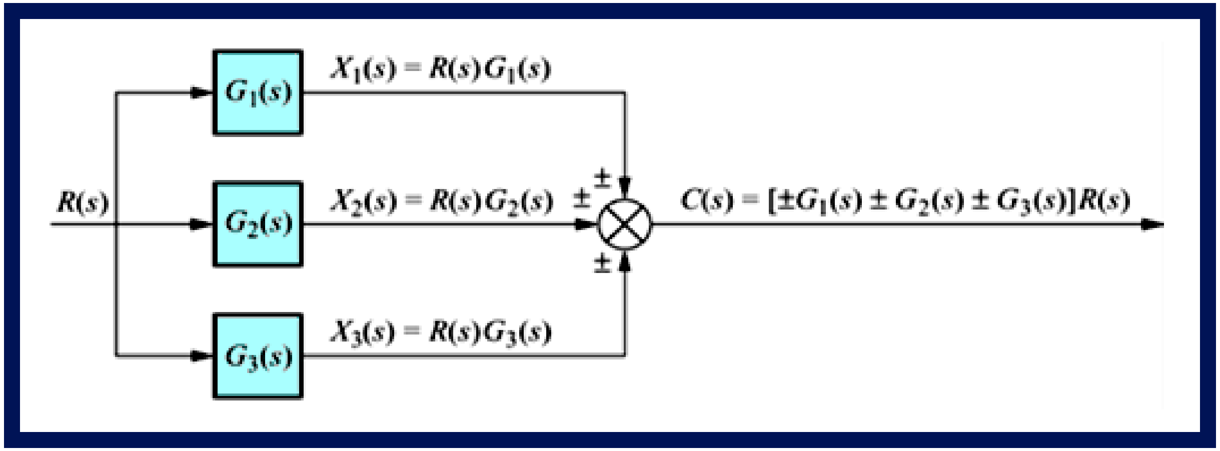
In each case, we start by drawing the signal nodes for that system.

Next we interconnect the signal nodes with system branches.

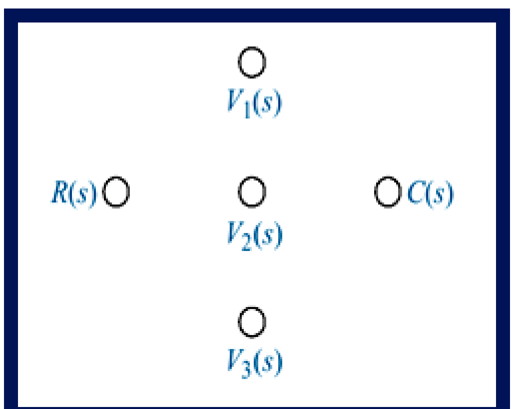


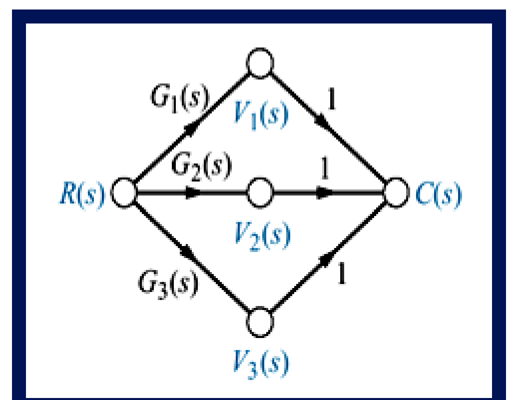


Example 1 (cont'd)

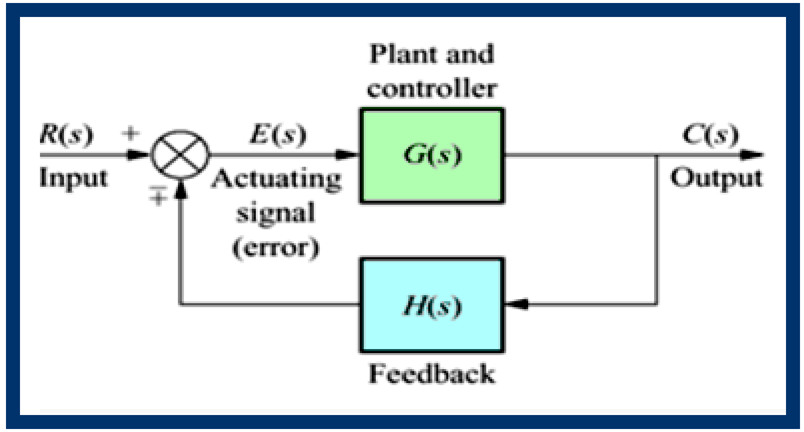


Solution

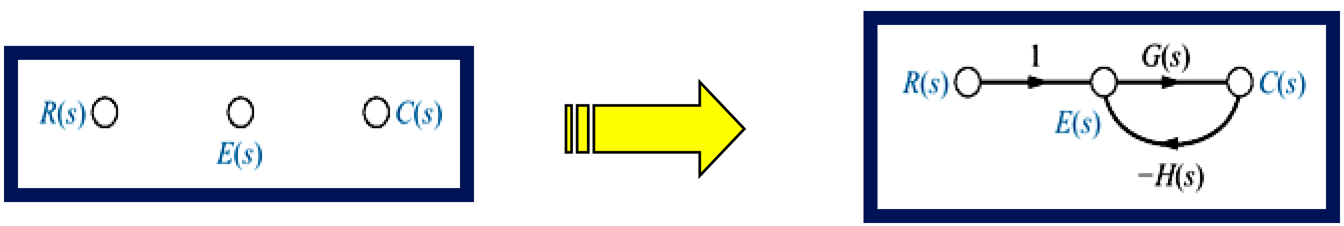




Example 1 (cont'd)

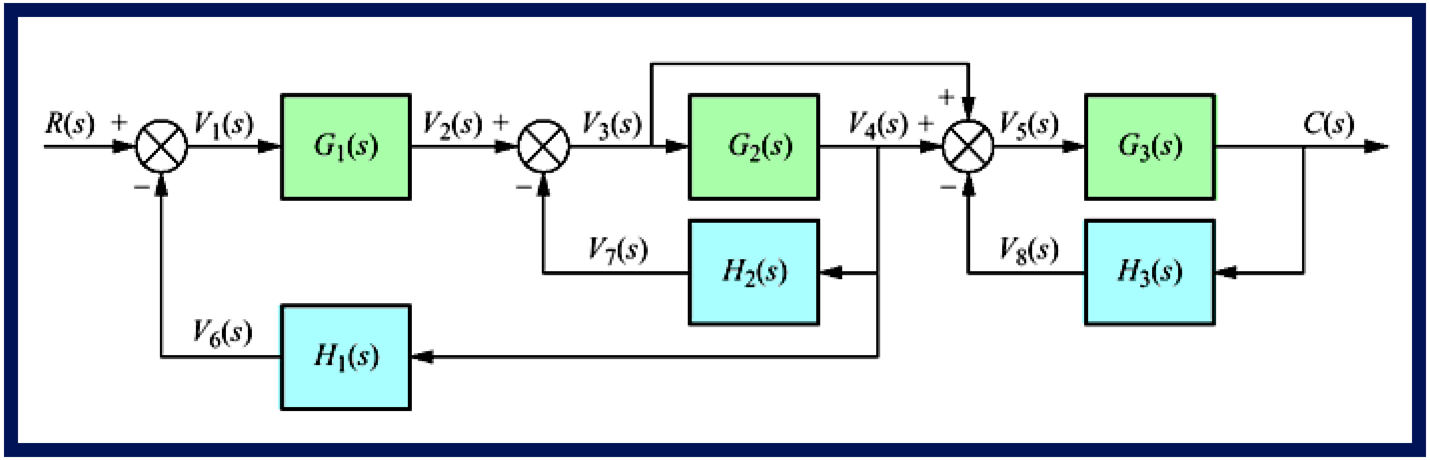


Solution



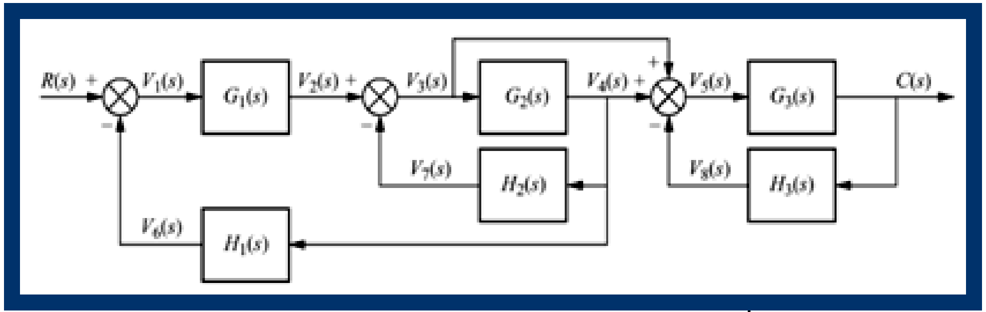
Example 2

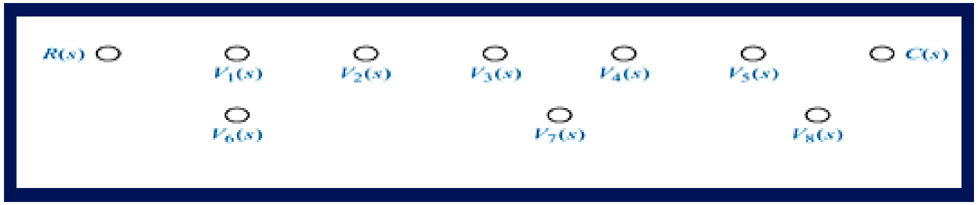
Convert the block diagram shown to a signal-flow graph.

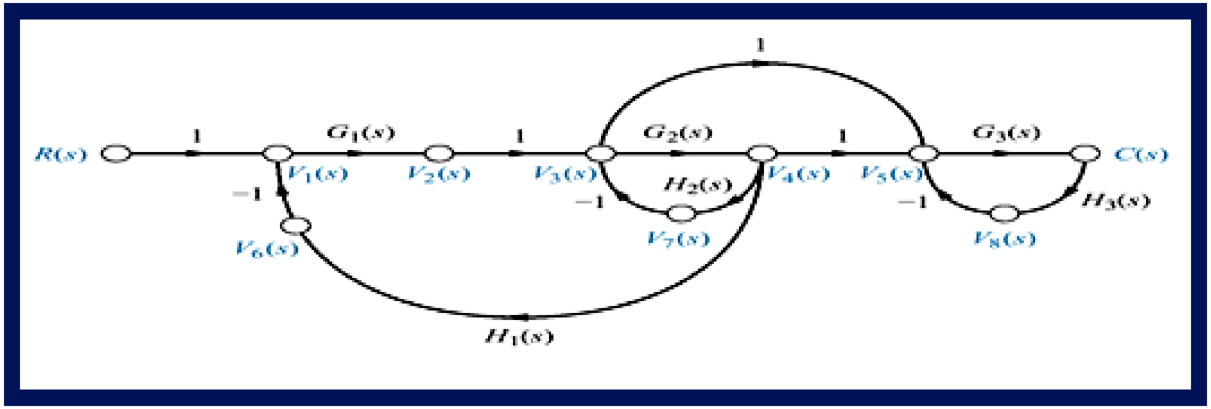


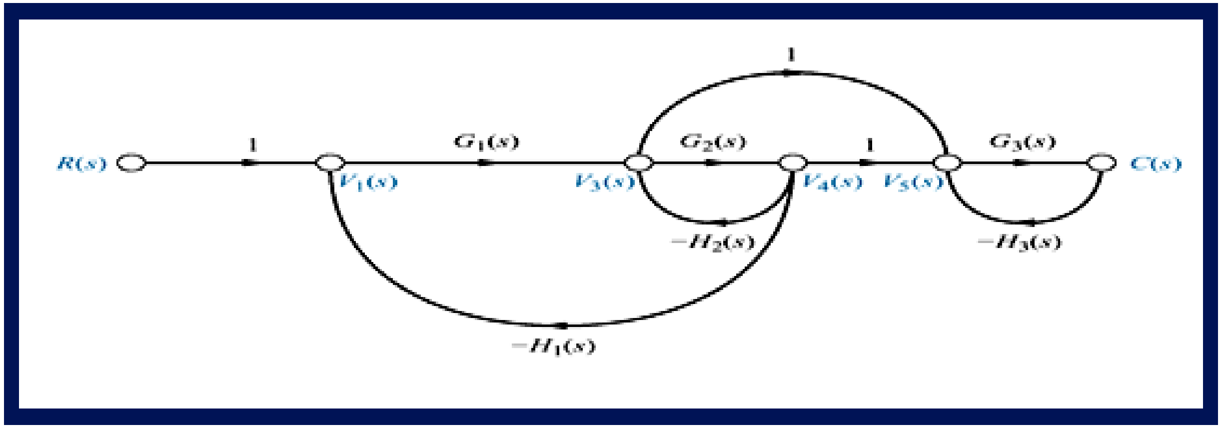
Example 2 (cont'd)

Solution









Mason's Rule

Mason's Rule

(Gain formula)

- Mason's rule or (Gain formula) is a technique used to reducing signal-flow graphs to single transfer function that relate the output of a system to its input.

Mason's Gain Formula

- A technique to reduce a signal-flow graph to a single transfer function requires the application of **one formula**.
- The transfer function, **C(s)/R(s),** of a system represented by a signal-flow graph is given by

$$G = \frac{1}{\Delta}\sum_{k=1}^{N} p_k\,\Delta_k = \frac{P_1\Delta_1 + P_2\Delta_2 + p_3\Delta_3 + \cdots + p_k\Delta_k}{\Delta}$$

P_k = *the k_{th} forward path gain*

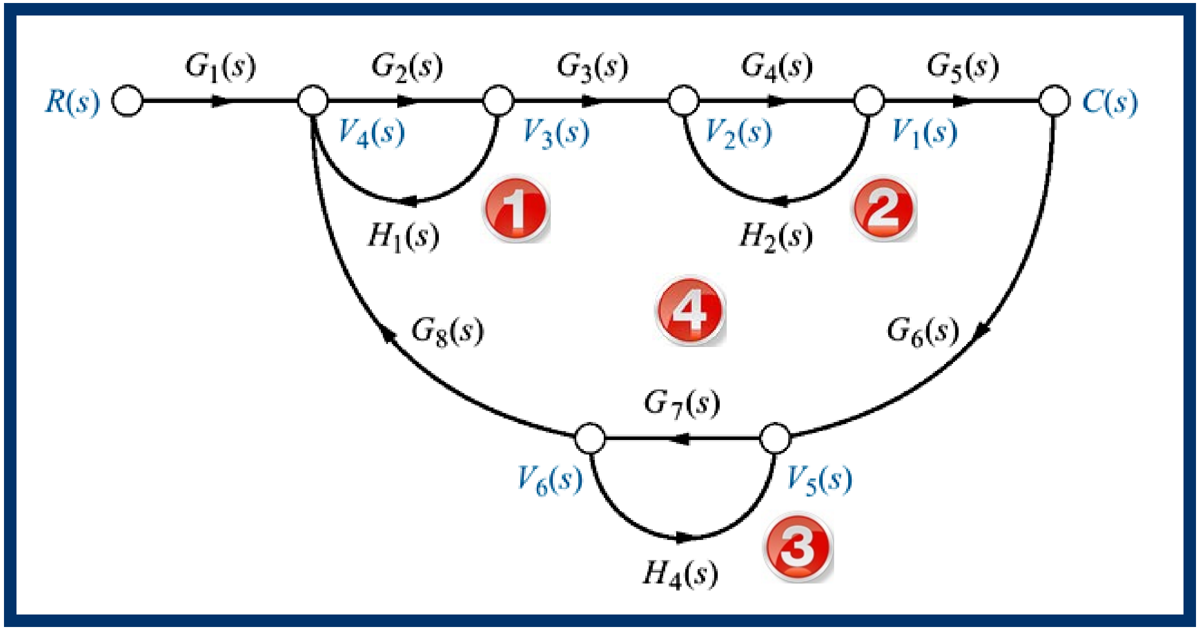
N = *total number of forward paths*

Δ = 1 - *(sum of the gains of **all** loops)* + *(sum of products of gains of all possible combinations of **two** non touching loops)* - *(sum of products of gains of all possible combinations of **three** non touching loops))+ so on .*

Δ_k = *1 – (loop-gain which does not touch the forward path)*

Example 3

Find the transfer function, *C(s)/R(s)* for the signal-flow graph shown below



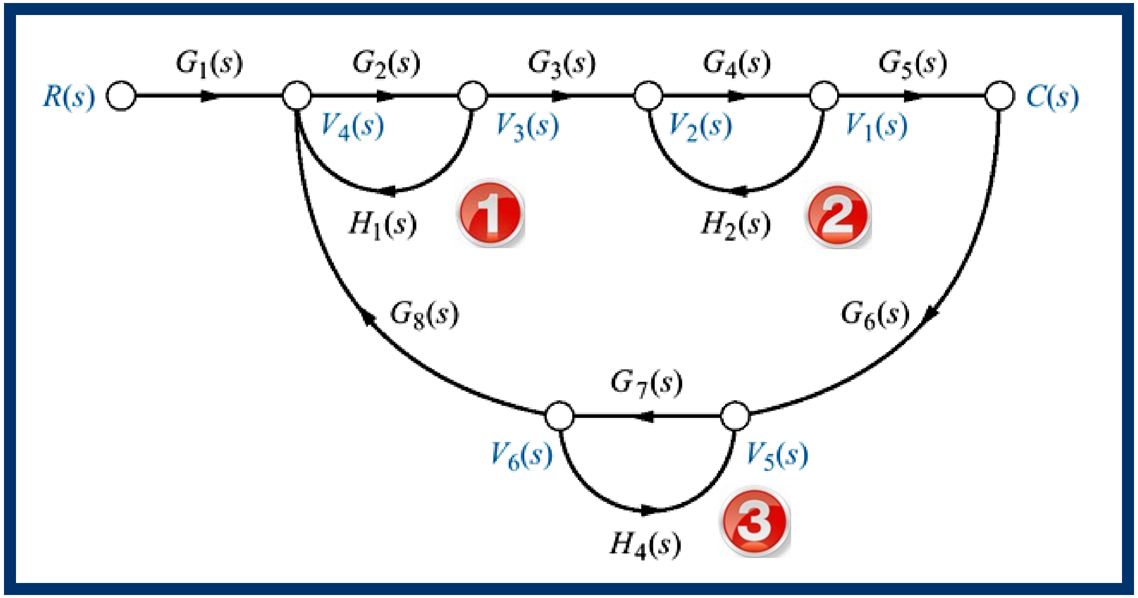
Solution

$N=1;$ $P_1=G_1G_2G_3G_4G_5$, Loops=4

1. $G_2(s)H_1(s)$ 2. $G_4(s)H_2(s)$ 3. $G_7(s)H_4(s)$

4. $G_2(s)G_3(s)G_4(s)G_5(s)G_6(s)G_7(s)G_8(s)$

Example 3 (cont'd)



Non touching loops taken **two** at time

Loop 1 and loop 2 : $G_2(s)H_1(s)G_4(s)H_2(s)$

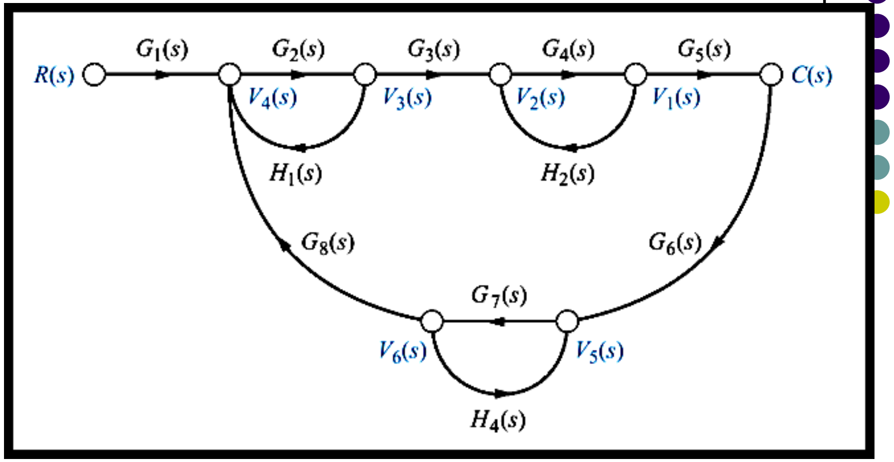
Loop 1 and loop 3 : $G_2(s)H_1(s)G_7(s)H_4(s)$

Loop 2 and loop 3 : $G_4(s)H_2(s)G_7(s)H_4(s)$

Non touching loops taken **three** at time

Loops 1, 2, and 3 : $G_2(s)H_1(s)G_4(s)H_2(s)G_7(s)H_4(s)$

Example 3 (cont'd)



$$G = \frac{1}{\Delta}\sum_{k=1}^{N} p_k \Delta_k$$

$$\Delta = 1 - [G_2(s)H_1(s) + G_4(s)H_2(s) + G_7(s)H_4(s) + G_2(s)G_3(s)G_4(s)G_5(s)G_6(s)G_7(s)G_8(s)]$$

$$+ [G_2(s)H_1(s)G_4(s)H_2(s) + G_2(s)H_1(s)G_7(s)H_4(s) + G_4(s)H_2(s)G_7(s)H_4(s)]$$

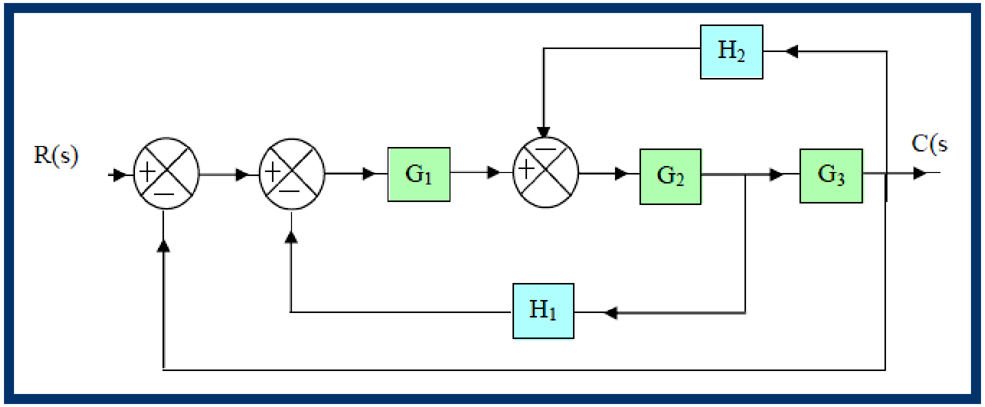
$$- [G_2(s)H_1(s)G_4(s)H_2(s)G_7(s)H_4(s)]$$

$$\Delta_1 = 1 - G_7(s)H_4(s)$$

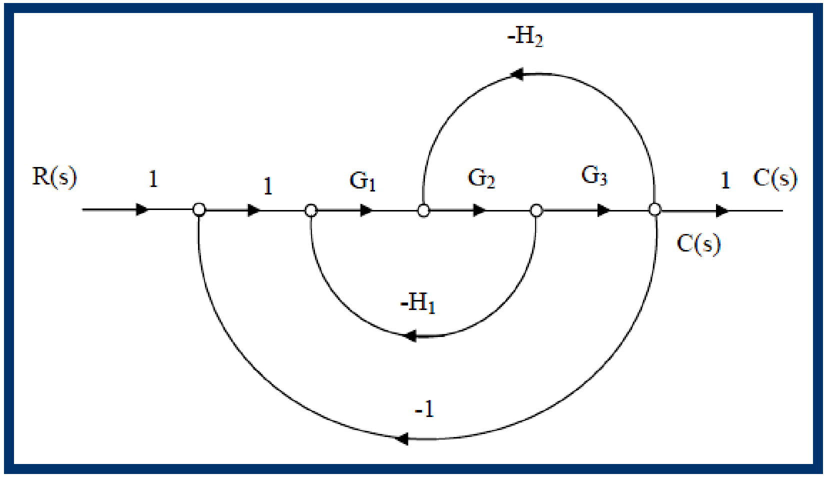
$$G(s) = \frac{[G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)][1 - G_7(s)H_4(s)]}{\Delta}$$

Example 4

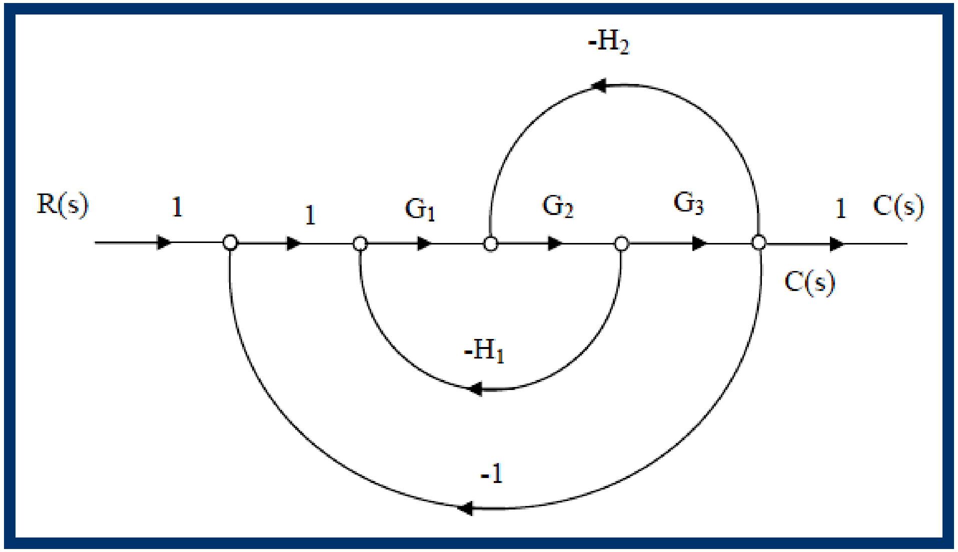
Find T.F C(s)/R(s) of the given block diagram using Mason's rule (Gain formula)



Solution



Example 4 (cont'd)



We have one forward path

$$P_1 = G_1 G_2 G_3$$

We have three loops

$$L_1 = -G_1 G_2 H_1$$

$$L_2 = -G_2 G_3 H_2$$

$$L_3 = -G_1 G_2 G_3$$

There are no non touching loops

All loops touching the forward path

$$\Delta = 1 - (L_1 + L_2 + L_3)$$

$$= 1 + G_1 G_2 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3$$

$$\Delta_1 = 1$$

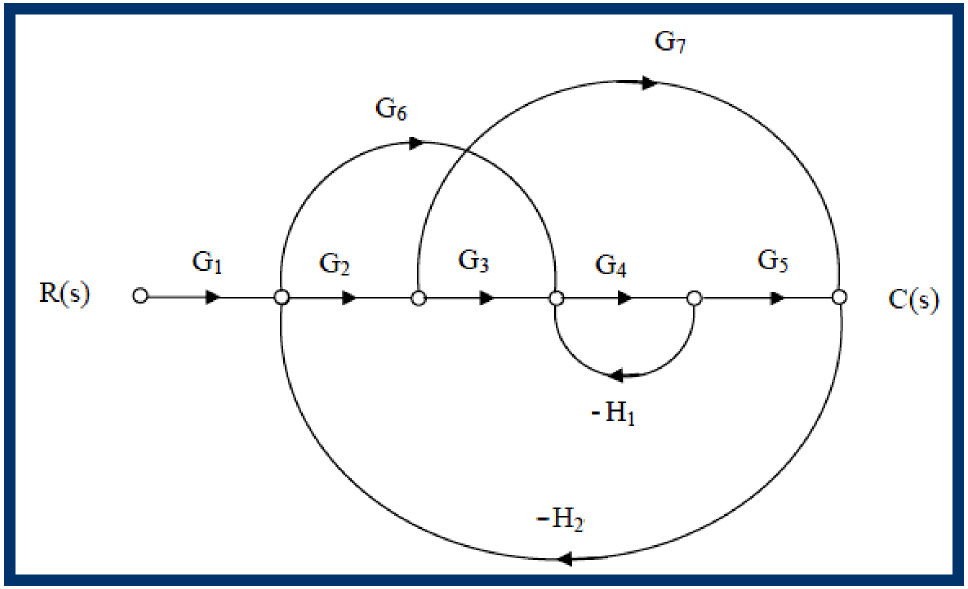
$$G = \frac{1}{\Delta} \sum_{k=1}^{N} p_k \Delta_k$$

$$\frac{C(s)}{R(s)} = G = \frac{P_1 \Delta_1}{\Delta}$$

$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3}{1 + G_1 G_2 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3}$$

Example 5

For the SFG shown un figure, Find the T.F $C(s)/R(s)$



Solution

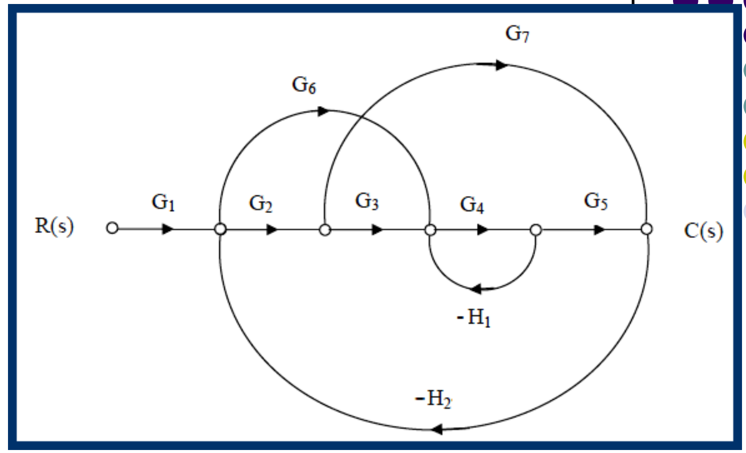
We have three forward path

$$P_1 = G_1G_2G_3G_4G_5$$
$$P_2 = G_1G_6G_4G_5$$
$$P_3 = G_1G_2G_7$$

We have three loops

$$L_1 = -G_4H_1$$
$$L_2 = -G_2G_7H_2$$
$$L_3 = -G_6G_4G_5H_2$$
$$L_4 = -G_2G_3G_4G_5H_2$$

Example 5 (cont'd)



$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + L_1 L_2$$

$$\Delta = 1 + G_4 H_1 + G_2 G_7 H_2 + G_6 G_4 G_5 H_2 + G_2 G_3 G_4 G_5 H_2 + G_2 G_4 G_7 H_1 H_2$$

All loops touching the forward paths P_1 ⇒ $\Delta_1 = 1$

All loops touching the forward paths P_2 ⇒ $\Delta_2 = 1$

Only loop L_1 non touching the forward path P_3 ⇒ $\Delta_3 = 1 + G_4 H_1$

$$G = \frac{1}{\Delta} \sum_{k=1}^{N} p_k \Delta_k$$

⇒

$$G = \frac{C(s)}{R(s)} = \frac{P_1 \Delta_1 + P_2 \Delta_2 + p_3 \Delta_3}{\Delta}$$

$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 G_4 G_5 + G_1 G_6 G_4 G_5 + G_1 G_2 G_7 + G_1 G_2 G_4 G_7 H_1}{1 + G_4 H_1 + G_2 G_7 H_2 + G_6 G_4 G_5 H_2 + G_2 G_3 G_4 G_5 H_2 + G_2 G_4 G_7 H_1 H_2}$$

Summary

– Signal flow graphs elements

– Signal flow graphs algebra

– Mason’s rule (Gain formula) for SFG

- **Next Lecture:**

– Time domain analysis of control systems